Chapter 6

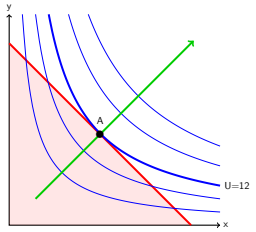
1. Production Functions
2. Marginal and Average Product
3. Isoquant and Marginal Rate of Technical Substitution
4. Elasticity of Substitution
5. Important Production Functions
6. Returns to Scale
7. Technological Progress
8. Comparison to Consumer Preferences

Comparison of Demand vs. Supply Framework

- Chapters 3-5: Demand Curve Framework:
  - Maximize subject to

$$\max_{x,y} U(x,y)$$

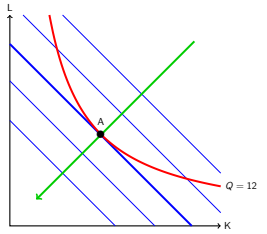
subject to $xP_x + yP_y \leq I$



- Chapters 6-8: Supply Curve Framework:
  - Minimize subject to

$$\min_{K,L} KP_K + LP_L$$

subject to $f(K,L) = Q$



Production Function

Definition (Production Function)

A mathematical representation that shows the maximum quantity of output a firm can produce given the quantities of inputs that it might employ.

- Inputs - Factors of Production
  - Capital (K)
  - Labor (L)
- Output - amount of good produced.
- **Production Set** - The set of technically feasible combinations of inputs and outputs.
- **Technically efficient** - The set of points in the production set at which the firm is producing as much output as it possibly can, given the amount of labor it employs.

Production Function Definitions

Total Product: The total amount produced.

Average Product: The average output per unit of input, i.e.

$$AP_L = \frac{\text{total product}}{\text{quantity of labor}} = \frac{Q}{L}$$

Marginal Product: Rate that output increases when the input is increased, i.e.

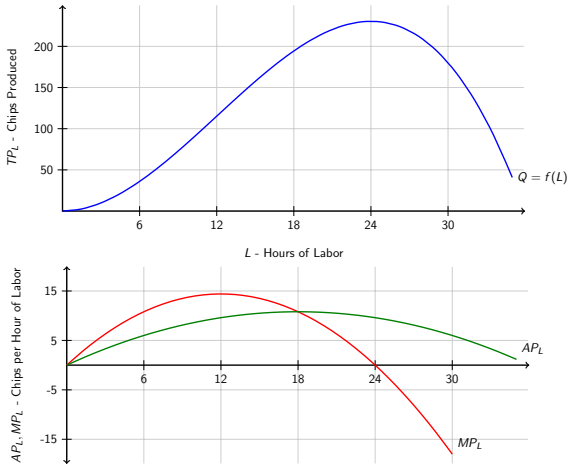
$$MP_L = \frac{\text{change in total product}}{\text{change in quantity of Labor}} = \frac{dQ}{dL}$$

(De)Increasing Marginal Returns: Levels of input such that the marginal product is (de)increasing.

- If $L: 1 \to 2$ then $Q: 10 \to 20$
- If $L: 2 \to 3$ then $Q: 20 \to$

Diminishing Total Returns: When total production decreases with additional inputs.

Production Functions - Graphically



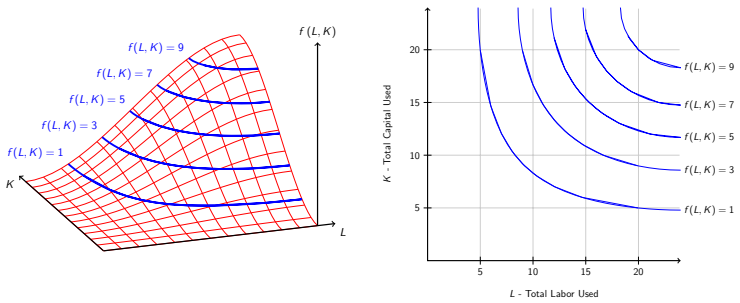
- Production Set:
- Technically efficient:
- Increasing Marginal Returns:
- Decreasing Marginal Returns:
- Decreasing Total Returns:

Production Functions

- When $MP < 0$ then TP is _____
- AP is maximized when _____
  - AP _____ if $AP < MP$.
  - AP _____ if $AP > MP$.
- **Law of diminishing marginal return:** As the usage of one input increases beyond some point, all else fixed, the marginal product of the variable input will decrease.
  - Compare with law of diminishing marginal utility.

Production Function with Multiple Inputs

- Many production functions have more than one input.



- **Isoquant** - A curve that shows all of the combinations of labor and capital that can _____

Production Function with Multiple Inputs

- Marginal Product of Labor,

$$MP_L = \frac{\text{change in}}{\text{change in}}$$

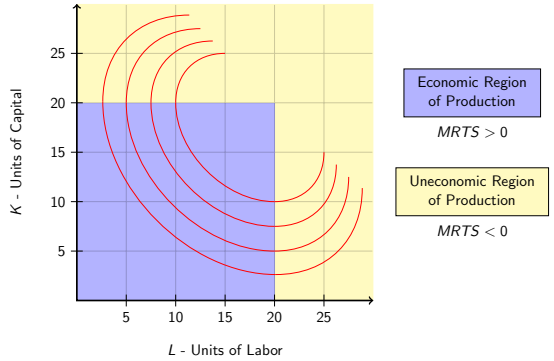
- Marginal Product of Capital,

$$MP_K = \frac{\text{change in}}{\text{change in}}$$

- **Marginal Rate of Technical Substitution:** The rate at which the quantity of capital can be reduced for every one unit increase in the quantity of labor, holding quantity of output constant.

$$MRTS_{L,K} =$$

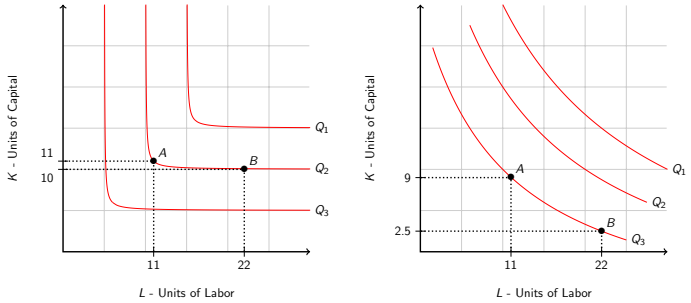
Isoquants



- **Diminishing MRTS:** A feature of production function in which the MRTS diminishes.

Substitutability Among Inputs

- How easily can firms substitute inputs?



- Large increase in L leads to small decrease in $K \Rightarrow$ Not substitutable.
- Large increase in L leads to large decrease in $K \Rightarrow$ Substitutable.

Elasticity of Substitution

- **Capital-Labor Ratio:** Ratio of Capital to Labor, $\frac{K}{L}$.
- **Elasticity of Substitution (σ):** A measure of how easy it is for a firm to substitute labor for capital.
  - Gives us measure of firm's input substitution opportunities.

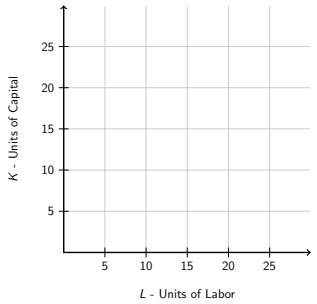
$$\sigma = \frac{\text{percentage change in capital-labor ratio}}{\text{percentage change in } MRTS_{L,K}} = \frac{\%\Delta\left(\frac{K}{L}\right)}{\%\Delta MRTS_{L,K}}$$

- σ is _____ if $MRTS_{L,K}$ is diminishing.
- If σ is _____, then there is little opportunity to substitute between inputs.
- If σ is _____, then there is substantial opportunity to substitute between inputs.

Linear Production Function

- **Linear Production Function:**

$$Q = aL + bK$$

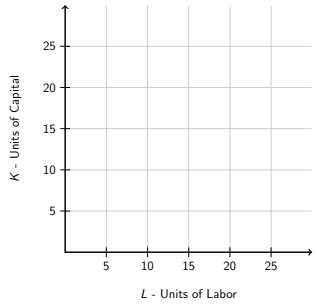


- **Elasticity of Substitution:**

Fixed-Proportion Production Function

- **Fixed-Proportion Production Function:**

$$Q = \min\{aL, bK\}$$

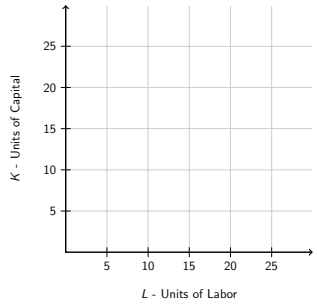


- **Elasticity of Substitution:**

Cobb-Douglas Production Function

- **Cobb-Douglas Production Function:**

$$Q = AL^{\alpha}K^{\beta}$$

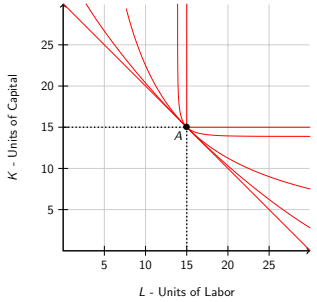


- **Elasticity of Substitution:**

Constant Elasticity of Substitution Production Function

- **CES Production Function:**

$$Q = \left[aL^{\frac{\sigma-1}{\sigma}} + bK^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$



- **Elasticity of Substitution:**

Returns to Scale

- **Returns to Scale:** Percentage increase in output compared to percentage increase of all inputs.

$$\text{Returns to Scale} = \frac{\%\Delta(\text{quantity of output})}{\%\Delta(\text{quantity of all inputs})}$$

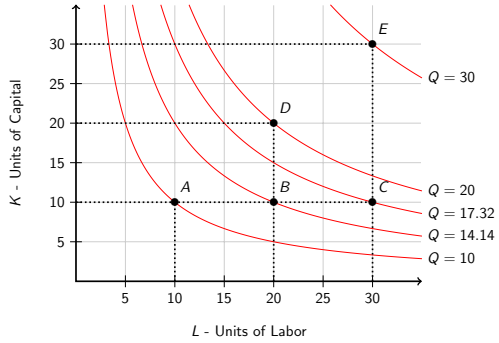
Decreasing Returns to scale:

Constant Returns to scale:

Increasing Returns to scale:

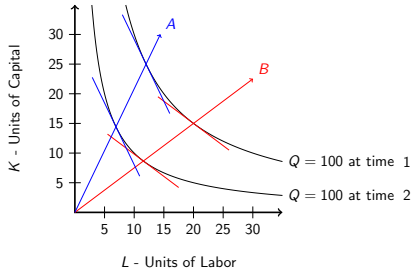
Diminishing Marginal Return vs. Returns to Scale

- Example: $f(L,K) = \sqrt{KL}$.



Technological Change

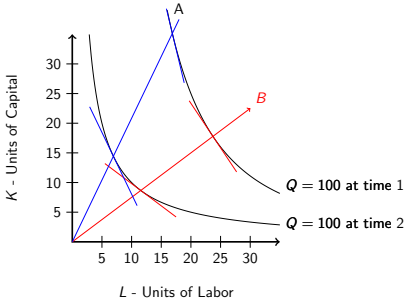
- **Technological Progress:** Change that enables firm to acheive more output from a given combination of inputs.
- **Neutral Technological Change:** Tech. progress that decreases the amount of inputs needed to produce a given output, without affecting $MRTS_{L,K}$.



- Along any ray from (0,0), $MRTS_{L,K}$ is the same.

Labor-Saving Technological Change

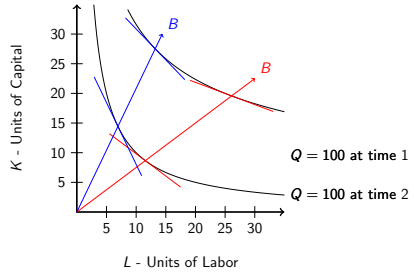
- **Labor-Saving Technological Change:** Tech. progress that causes the MP_K to increase relative to the MP_L.



- Along any ray from (0,0), $MRTS_{L,K}$ decreases (flatter) with technological progress.

Capital-Saving Technological Change

- **Capital-Saving Technological Change:** Tech. progress that causes the MP_L to increase relative to the MP_K.



- Along any ray from (0,0), $MRTS_{L,K}$ increases (steeper) with technological progress.

Exercise

	Time 1	**Time 2**
Production Function	$Q^1 = \sqrt{KL}$	$Q^2 = L\sqrt{K}$

- Verify that the firm experiences technological progress from Time 1 to Time 2.
- What type of technological progress is this (neutral, labor-saving, capital-saving)?

Comparison with Utility

- Comparison of Consumers and Producers

	Consumer	**Producer**
Inputs		
Output Function		
Change in inputs		
2D Representation		
Substitution		