1. Suppose the production of Toyota cars is characterized by a Cobb-Douglas production function: $Q = 50L^{2/3}K^{1/3}$.

   a) Find the cost-minimizing combination of labor and capital if Toyota wants to
   - produce $Q_0 = 2000$ cars, and faces
   - a cost of labor of $w = 1$ per unit and
   - a cost of capital of $r = 4$ per unit.

   b) Find the capital and labor demand curves. (In terms of Q_0, w, r)

   c) Find the total cost function. (In terms of Q_0, w, r)

2. The processing of payroll for the 100 workers in a firm can either be done using 1 hour of computers time (denoted by K) and no clerks or with 10 hours of clerical time (denoted by L) and no computing time. Computers and clerks are perfect substitutes; for example, the firm could also process its payroll using 1/2 hour of computer time and 5 hours of clerical time.

   a) Sketch the isoquant that shows all combinations of clerical time and computer time that allows the firm to process the payroll for 100 workers.

   b) Suppose computer time costs \$2 per hour and clerical time costs \$3 per hour. What are the cost minimizing choices of L and K? What is the minimized total cost of processing the payroll?

   c) Suppose the price of clerical time remains at \$3 per hour. How high would the price of an hour of computer time have to be before the firm would find it worthwhile to use only clerks to process the payroll?

3. A plant's production function is $Q = 3KL + 6K$. The price of labor and services w is \$3 and of capital services r is \$7 per unit. The firm has a production target of $Q_0 = 63$.

   a) What are the marginal product functions, MP_L and MP_K, for this production function?

   b) How much labor and capital would the firm use to minimize cost in the long-run?

   c) How much labor and capital would the firm use to minimize cost in the short-run if they are stuck with $\bar{K} = 6$ units of capital?

   d) How much money is the firm sacrificing by not having the ability to choose its level of capital optimally? (Compare optimal short-run to long-run total cost)

4. The following incomplete table shows a firm's various costs of producing up to 6 units of output. Fill in as much of the table as possible. If you cannot determining the number in a box, explain why it is not possible to do so.

Q	TC	TVC	AFC	AC	MC	AVC	TFC
1				**100**			
2		**50**	**30**				
3					**40**		
4						**30**	
5							
6	**330**				**80**		

5. A firm's long-run total cost curve is $TC(Q) = 100Q - 16Q^2 + 3Q^3$.

   a) What is the firm's long-run marginal cost curve?

   b) Over what range of output does the production function exhibit economies of scale?

   c) Over what range of output does the production function exhibit diseconomies of scale?

6. A firm produces a product with labor and capital as inputs. The production function is described by $Q = \sqrt{LK}$. Let $w = 1$ be the price of labor and $r = 1$ be the price of capital.

   a) What are the marginal product functions, MP_L and MP_K?

   b) Find the equation for the firm's long-run total cost curve as a function of quantity Q. (Hint: first you have to find the input demand functions)

   c) Solve the firm's short-run cost-minimization problem when capital is fixed at a quantity of 4 units (i.e. $\bar{K} = 4$). derive the equation for the firm's short-run total cost curve as a function of quantity Q and graph it together with the long-run total cost curve.

7. Tricycles must be produced with 3 wheels and 1 frame for each bicycle. Let Q be the number of tricycles, W be the number of wheels, and F be the number of frames. The price of a wheel is P_W and the price of a frame is P_F.

   a) What is the long-run total cost function for producing tricycles, $TC(Q, P_W, P_F)$? (Hint: think of this intuitively rather than using formulas)

   b) What is the production function for tricycles $Q(F, W)$? (This tells how many tricycles can be produced for each combination of F and W)